

Separation of
Variables

$$\frac{dy}{dx} = (xy)^2$$

$$\frac{dy}{dx} = x^2 y^2$$

$$\int \frac{dy}{y^2} = \int x^2 dx$$

$$-\frac{1}{y} + c' = \frac{x^3}{3} + c$$

$$-\frac{1}{y} = \frac{x^3}{3} + c$$

$$\frac{1}{y} = -\frac{x^3}{3} + c$$

$$y = -\frac{3}{x^3} + c$$

~~Fail~~ Succeed with

$$\textcircled{1} \frac{dy}{dx} = \frac{2x}{y}$$

$$y = \pm \sqrt{2x^2} + C$$

$$= \pm \sqrt{2} x + C$$

$$\textcircled{2} \frac{dy}{dx} = \frac{5x}{y}$$

$$= \pm \sqrt{5x^2} + C$$

$$= \pm \sqrt{5} x + C$$

$$\textcircled{3} \frac{dy}{dx} = \sqrt{x} y$$

$$y = C e^{\frac{2}{3} x^{3/2}}$$

$$\int \frac{dy}{y} = \int \sqrt{x} dx$$

$$\ln|y| = \frac{2}{3} x^{3/2} + C$$

$$y = e^{\frac{2}{3} x^{3/2} + C}$$

$$y = e^{\frac{2}{3} x^{3/2}} \cdot e^C$$

$$y = C e^{\frac{2}{3} x^{3/2}}$$

$$\textcircled{4} \frac{dy}{dx} = ky$$

$$y = C e^{kx}$$

$$\int \frac{dy}{y} = \int k dx$$

$$\ln|y| = kx + C$$

$$y = e^{kx + C}$$

$$y = C e^{kx}$$

exponential function

$$\frac{dy}{dt} = ky$$

$$y = Ce^{kx}$$

the rate of growth or decay is directly proportional
to the amount of stuff you have

Sect. 6.4

1-10 (5), 11, 14, 15, 18, 19, 22, 23

$$\frac{dy}{dx} = \frac{x}{y}$$

$$\int y dy = \int x dx \rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C \leftarrow$$

$$\frac{2^2}{2} = \frac{1^2}{2} + C$$

$$2 = \frac{1}{2} + C = \frac{3}{2}$$

$$y^2 = x^2 + C$$

$$2^2 = 1^2 + C \quad C = 3$$

$$y^2 = x^2 + 3$$

$$y = \pm \sqrt{x^2 + 3}$$